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LETTER TO THE EDITOR

The parton model, electromagnetic form factors of hadrons and high energy elastic $\pi^{\pm}$p scattering

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Abstract. Following the idea of Wu and Yang that the electromagnetic and nuclear distributions behave similarly at large momentum transfer, we have studied the differential cross section for $\pi^{\pm}$p elastic scattering at high energy within the framework of the parton model. By using the idea of Drell and Yan we have calculated the electromagnetic form factors of the pion and proton and they are shown to decrease as $t^{-1/2}$ and t^{-2} respectively at large t.

Recent studies of deep inelastic lepton–hadron scattering have given us a fruitful way of studying the structure of elementary particles. It is expected that this process can reveal the secret of hadrons. The quark version of the parton model has been widely investigated by many authors. They have considered the constituents of the proton to be three 'valence' quarks of spin $\frac{1}{2}$ and an infinite number of quark–antiquark pairs. However this was not very successful. In fact, Gardiner and Majumdar (1970) have pointed out that the experimental results indicate that partons are integrally charged rather than fractionally charged and that most of the partons in the proton are neutral. What role do these neutral particles play in the proton? Several authors take these as gluons which bind quarks, but this idea is not appealing because of the large number of neutral particles that are required to interpret the νW_2 behaviour. In a recent paper by Bandyopadhyay *et al* (1972a) it is shown that we can have a model for partons consistent with the experimental data in which the number of partons participating in the deep inelastic scattering is finite and there is practically no contribution from the cloud. A general consequence of this, however, is that νW_2 should tend to zero as $\omega \to \infty$. This is to be tested experimentally in the high energy region and is yet to be proved. However, our model gives direct support to the conjecture of Bloom and Gilman (1970) that resonances do contribute to the scaling behaviour and there is no diffraction contribution (ie pomeronchukon exchange) in the scaling limit curve. It is evident that this is in full agreement with the concept of duality.

Considering our parton model, we wish to study the Wu–Yang conjecture regarding the differential cross section for elastic $\pi^{\pm}$p scattering.

From parton model considerations, it can be shown (Bjorken and Paschos 1969) that the inelastic structure function $W_2(\nu, Q^2)$ of lepton–nucleon scattering is given by

$$\nu W_2(\nu, Q^2) = \Sigma P(N)\left\langle \sum_1^N Q_i^2 \right\rangle x f_N(x) \tag{1}$$

where $x = Q^2/2M\nu = 1/\omega$, ν is the invariant energy transfer and $-Q^2 = q_t^2 = t$ is the squared four-momentum transfer. $P(N)$ is the probability of finding N partons in the proton, $\langle Q_i^2 \rangle_N$ is the average value of the sum of the squared charges of the partons in a configuration of N partons and $f_N(x)$ is the probability of finding a parton with a longitudinal fraction x of the protons momentum. Bjorken and Paschos (1969) have discussed the property of the function $f_N(x)$ and they derived the following function for this:

$$f_N(x) = (N-1)(1-x)^{N-2}. \tag{2}$$

To obtain the structure function νW_2 for a meson and a baryon, we apply our parton model to the proton and pion as we are interested only in π^+p and π^-p elastic scattering, and get

$$\nu W_2 = \Sigma P_{\mathrm{p}}(N) \langle Q_i^2 \rangle_N x f_N(x) \qquad \text{for proton} \tag{3}$$

$$= \Sigma P_\pi(N) \langle Q_i^2 \rangle_N x f_N(x) \qquad \text{for pion} \tag{4}$$

where $P_{\mathrm{p}}(N)$ and $P_\pi(N)$ are the probabilities of finding N partons in the proton and pion respectively.

In our parton model the proton consists of five partons. One of the partons in the proton is positively charged and others are neutral. It has been shown (Bandyopadhyay *et al* 1972b) that in our model the π meson can be obtained from two partons having particle and antiparticle relations. According to that model for the pion any two of the partons (having particle and antiparticle relations) can create a pion in the proton. Using that idea, we have shown that the behaviour of the differential cross section for γp $\rightarrow \pi$p . . . etc can be explained nicely (Bandyopadhyay *et al* 1972b). Using our model, we get

$$\nu W_2{}^{\mathrm{p}} = 4x(1-x)^3 \qquad \text{for proton} \tag{5}$$

$$\nu W_2{}^{\pi} = x \qquad \text{for pion.} \tag{6}$$

It is interesting to compare (5) and (6) with a conjecture of Drell and Yan (1970) who showed that the exponent of the asymptotic behaviour of νW_2 near $x = 1$,

$$\nu W_2 \sim x(1-x)^n, \tag{7}$$

is correlated with the asymptotic q^2 dependence of the electromagnetic elastic form factor $F_1(q^2)$

$$F_1(q^2) \sim \left(\frac{-1}{q^2}\right)^{(n+1)/2}. \tag{8}$$

Using the conjecture of Drell and Yan, we can write the electromagnetic elastic form factor

$$F_{\mathrm{p}}(q^2) \sim \left(\frac{-1}{q^2}\right)^2 \qquad \text{for proton} \tag{9}$$

$$F_\pi(q^2) \sim \left(\frac{-1}{q^2}\right)^{1/2} \qquad \text{for pion.} \tag{10}$$

In order to normalize our form factor so that $F_{\mathrm{p},\pi}(0) = 1$ and for all q^2, we take the formula

$$F_{\mathrm{p}}(q^2) = \frac{1}{(1-q^2/m_\rho{}^2)^2} \qquad \text{for proton} \tag{11}$$

which is a conventional formula, and

$$F_\pi(q^2) = \frac{1}{\{1-(q^2/9m_\pi{}^2)\}^{1/2}} \qquad \text{for pion.} \tag{12}$$

We have tentatively taken this form factor for the pion in order to fit the observed pion charge radius $r_\pi = 0{\cdot}86 \pm 0{\cdot}09\,F$ (Mistretta *et al* 1968). If we use equation (12) then we get $r_\pi = (-6F'(0))^{1/2} = 0{\cdot}83F$. The form factor for the pion falls much slower than the proton form factor. Equation (12) is in excellent agreement with the recent experimental results (Coward *et al* 1968, Brown *et al* 1971) shown in figure 1.

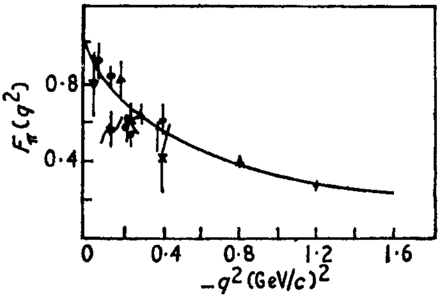

Figure 1. Pion form factor $F_\pi(q^2)$ against $-q^2$, the square of the momentum transfer. The full curve is our fit; the data are from experiments referred to in the text.

Because of this it is encouraging to study elastic $\pi^\pm$p scattering at high energy. The differential cross section of hadron–hadron scattering (neglecting multiple scattering inside the particle) is related by the formula

$$\lim_{s\to\infty} \frac{d\sigma_{AB}}{dt} \propto (F_A(t)F_B(t))^2 \tag{13}$$

where $F_A(t)$, $F_B(t)$ are the electromagnetic form factors of the A and B particles. This relation has been suggested by Wu and Yang (1965) considering the hadron as an extended object at large momentum transfer. Abarbanel *et al* (1968, 1969) have suggested an even more specific mechanism for this large momentum transfer region, in which the scattering occurs owing to an effective four fermion vector (or axial vector) coupling with the universal form factor $G_M{}^4(q^2)$ for pp elastic scattering. Assuming the same idea that the interaction between partons is the effective four fermion vector (or axial vector) coupling, we can get the same relation (13) if we consider in our parton model that the interacting partons have almost all the momenta of the parent hadrons. In our case, the differential cross section for $\pi^\pm$p elastic scattering can be written as

$$\frac{d\sigma(\pi^\pm p)}{dt} = \left.\frac{d\sigma(\pi^\pm p)}{dt}\right|_{t=0} F_\pi{}^2(t)F_p{}^2(t) \tag{14}$$

using equations (11) and (12), we compare equation (14) with the corresponding experimental data (Orier *et al* 1966) which are shown in figures 2 and 3. We have taken the value

$$\left.\frac{d\sigma(\pi^\pm p)}{dt}\right|_{t=0} = 35\,\text{mb}\,(\text{GeV}/c)^{-2}. \tag{15}$$

Here we have presented only that figure for the differential cross section for $\pi^{\pm}p$ elastic scattering neccessary to study our electromagnetic form factors. From figures 2 and 3 we see that the agreement is very good up to $|t| < 1(\mathrm{GeV}/c)^{-2}$ for all energies, although the dip structure appears for low energy at $|t| = 0{\cdot}6\,(\mathrm{GeV}/c)^2$ and as the incident energy increases the dip structure disappears. But at $|t| = 3(\mathrm{GeV}/c)^2$

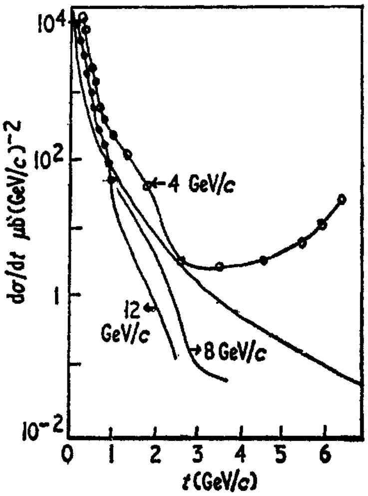

Figure 2. $d\sigma/dt(\pi^+p)$ against t at various P_L. The full curve (nonmentioned energy) is our theoretical curve, the data are from experiments referred to in the text.

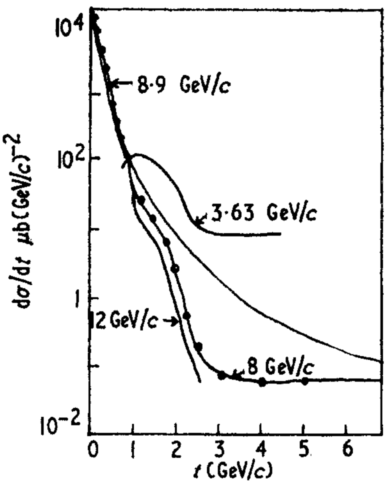

Figure 3. $d\sigma/dt(\pi^-p)$ against t at various P_L. The full curve (nonmentioned energy) is our theoretical curve, the data are from experiments referred to in the text.

again the dip structure appears, which seems to be due to multiple scattering between partons. It is expected that at large t, the model of Wu and Yang must break down, because there may be multiple scattering effects. We will not discuss multiple scattering between partons. We have seen that our model does not work well at large t, where the multiple scattering between particles seems to appear. This is explicitly included in an optical model developed by Chou and Yang (1968).

Again it is not yet clear whether the experimental dips at $|t| = 0{\cdot}6\ (\mathrm{GeV}/c)^2$ and near the $t = 3(\mathrm{GeV}/c)^2$ region can be explained by the optical model. We believe that the dip structure arises owing to the effect of the multiple scattering between the partons. It is to be noted here that the differential cross section for $\pi^{\pm}$p elastic scattering for large t decreases as t^{-5}.

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References

Abarbanel H D I, Drell S D and Gilman F J 1968 *Phys. Rev. Lett.* **20** 280
—— 1969 *Phys. Rev.* **177** 2458
Bandyopadhyay P, Raychaudhiri P and De S 1972a *Lett. Nuovo Cim.* **3** 43
—— 1972b submitted for publication
Bjorken J D and Paschos E A 1969 *Phys. Rev.* **185** 1975
Bloom E D and Gilman F J 1970 *Phys. Rev. Lett.* **25** 1140
Brown *et al* 1971 *Phys. Rev. Lett* **26** 991
Chou T T and Yang C N 1968 *Phys. Rev.* **175** 1832
Coward D H *et al* 1968 *Phys. Rev. Lett.* **20** 292
Drell S D and Yan T M 1970 *Phys. Rev. Lett.* **24** 181
Gardinar C W and Majumdar D P 1970 *Phys. Rev.* D **1** 151
Mistretta C *et al* 1968 *Phys. Rev. Lett.* **20** 1523
Orear J *et al* 1966 *Phys. Rev.* **152** 1162
Wu T T and Yang C N 1965 *Phys. Rev.* **137** B708